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PLANNER MODEL FOR ESTIMATING THE DYNAMIC OF EPIDEMIC SPREAD UNDER LIMITED RESOURCES

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Abstract The article proposes a mathematical formulation aimed at estimating the spread of the virus, while considering the limitations imposed to available resources for epidemic prevention. Mathematically, such a problem can be formulated as a constrained optimal control problem. An algorithm using the penalty function method to solve the problem is proposed. Computational experiments were carried out to simulate the development of the initial stage of the epidemic based on real epidemiological data on the Covid-19 pandemic in Novosibirsk in 2020.

Key words: optimal control problem with restrictions, Covid-19, penalty functions method.

AMS Mathematics Subject Classification: 93-10.

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1 Introduction

The coronavirus pandemic has highlighted a lot of problems, including the necessity for the formulation of strategies regulating social distancing policies by governments. The various approaches adopted by different countries to mitigate the spread of morbidity have demonstrated that these strategies yield diverse impacts on the socio-epidemiological situation, as illustrated by examples in [1]–[3]. In particular, it was shown that non-pharmaceutical interventions (virus prevention) are critical in the early stages of a new disease's emergence, given that the features of the spread of the virus remain uncertain when confronting a novel threat.

The social literature identifies several challenges faced by planners, when choosing a particular preventive strategy [4]–[6]. This include social factors, such as skepticism towards information provided by health authorities and a tendency to underestimate risk becoming infected, and also economic limitations when implementing quarantine measures. As a result, assessing possible scenarios for government decisions regarding the implementation of various antiviral measures in the context of resource constraints becomes increasingly relevant.

From the researcher's perspective, the problem involves identifying effective mathematical models that accurately represent the situation, along with methodologies for solving these models. Note, that several studies have been undertaken with similar objectives. For instance, in [7], an optimal control model is introduced, derived from the fundamental epidemiological SIS model. In this framework, the decision-maker utilizes collected tax revenues to allocate funds either for prevention measures aimed at the population or for the treatment of infected individuals. By quantifying the social costs associated with prevention and treatment, the authors ascertain which policies yield the greatest cost-effectiveness under varying conditions, demonstrating that prevention (or treatment) is advantageous when infection rates are low (or high). The article [8] presents a mean field model based on the assumption that individuals act in their self-interest to maximize personal utility. It was concluded that, given the costs involved in encouraging the population to adhere to antiviral measures, the policy of reducing contact among infected individuals should persist even after the epidemic has waned.

The authors of the paper [9] put forth an enhanced discretized SIR model, demonstrating that the prioritization of preventive measures by the population results in decreased consumption and an economic downturn, while simultaneously contributing to a reduction in the number of infections. Similarly, the studies [10]–[12] present extensions to differential epidemiological models of SIR, which incorporate the influence of morbidity spread on the economic metrics of the region being modeled.

Thus, the search for new models for assessing various anti-epidemic measures is an actual problem. Note that most of the work in this area is based on describing the dynamics of the spread of the disease using a simple epidemiological SIR model. At the same time, an assessment of the dynamics of the COVID-19 pandemic shows that the clustering into non-immune/susceptible (S), infected (I) and immune/recovered (R) groups is not enough to describe the spread of a complex virus. Therefore, the estimates obtained using such models remain very distant. Moreover, researchers in this area do not consider the limitations imposed on the available resources for the prevention and treatment of morbidity, which can also lead to an incorrect assessment of the consequences of the spread of the epidemic and the anti-epidemic measures used by the planner.

The current work presents a mathematical model of optimal control that simulates the dynamics of the spread of epidemic, taking into account limited resources aimed at combating the spread of the virus. Here, the behavior of the system is determined by the cost functional and restrictions, which can be chosen in a different form, taking into account the simulated situation. A general approach to solving such problems is considered.

2 Mathematical formulation of problem

2.1 The description of epidemic propagation

The basis for assessing the impact of various restrictive measures on the spread of morbidity is the method of describing the dynamics of the epidemic. Here, the generally accepted and most widespread approach is the mathematical SIR model and its derivatives. The first such model was proposed back in the 1920s and the general idea of this approach is to cluster the population into several epidemiological groups (in the case of the SIR model: S -susceptible/non-immune, I-infected; R - those who have received immunity due to recovery or death) and introduction of connections between them. The probabilities of an individual's transition from one group to another, as well as the initial distribution of the population among epidemiological groups, are the parameters of the model. Nowadays, SIR models (also called compartmental models) are widespread and there are more than a hundred modifications of the original model

[13], associated with more detailed clustering of the population and/or with taking into account the temporal variability of model parameters.

The mathematical model used in this study for description of viral dynamic is based on the following principles. Firstly, clustering of the population into 7 epidemiological groups was chosen as the basis for the epidemiological component of the model: $S(t)$ represents the portion of the population that is susceptible to the virus, $E(t)$ corresponds to individuals who are asymptomatically infected (exposed), $I(t)$ refers to those who exhibit symptoms of infection, $R(t)$ indicates individuals who have recovered, $H(t)$ encompasses hospitalized patients, $C(t)$ denotes those in critical condition, and $D(t)$ signifies the number of fatalities. The interaction among these groups is governed by probabilities of transferring (model coefficients) as illustrated in Fig. 1. Mathematically, the SEIR-HCD model is presented as the following system of ordinary differential equations.

$$\begin{cases} \dfrac{d\mu_S}{dt} = -(1-a/5)\left(\beta_I(t)\mu_S(t)\mu_I(t) + \beta_E(t)\mu_S(t)\mu_E(t)\right) + w_{imm}\mu_R(t), \\ \dfrac{d\mu_E}{dt} = (1-a/5)\left(\beta_I(t)\mu_S(t)\mu_I(t) + \beta_E(t)\mu_S(t)\mu_E(t)\right) - w_{inc}\mu_E(t), \\ \dfrac{d\mu_I}{dt} = w_{inc}\mu_E(t) - w_{inf}\mu_I(t), \\ \dfrac{d\mu_R}{dt} = \theta \cdot w_{inf}\mu_I(t) + (1-\varepsilon_{HC}) \cdot w_{hosp}\mu_H(t) - w_{imm}\mu_R(t), \\ \dfrac{d\mu_H}{dt} = (1-\theta) \cdot w_{inf}\mu_I(t) + (1-m) \cdot w_{crit}\mu_C(t) - w_{hosp}\mu_H(t), \\ \dfrac{d\mu_C}{dt} = \varepsilon_{HC} w_{hosp}\mu_H(t) - w_{crit}\mu_C(t), \\ \dfrac{d\mu_D}{dt} = \mu w_{crit}\mu_C(t) \end{cases} \tag{1}$$

with corresponding initial values: $\{\mu_i(0)\}$, $i \in \{S, E, I, R, H, C, D\}$

$$\mu_i(0) = \mu_{0i} = const. \tag{2}$$

Here μ_i represents the proportion of the population associated with the corresponding epidemiological group. A comprehensive description of the parameters along with their potential ranges of variation is provided in Table 1. Unlike the model discussed in [15], we introduce the fraction of each epidemiological group at a given time t rather than the absolute number of individuals in each group. Additionally, in contrast with [15] we define the model parameters as frequencies w_{imm}, w_{inc}, w_{inf}, w_{hosp}, w_{crit}, where $w_\circ = \frac{1}{t_\circ}$ with the corresponding values for each parameter denoted by $\circ$.

The SEIR-HCD model has more detailed description of epidemic spread than the basic SIR model. As an advantage, the model takes into account the hospitalized and critically ill people, the treatment of whom is more expensive in economic sense. The model is studied in [15, 17] in relates to sensitivity and prognoses possibilities for coronavirus. In the paper [18] the agent model based on (1) is presented.

2.2 Economic correction to model

Now add the economic component to the model. Assume that the total income available the social planner for carrying out anti-epidemic measures (prevention and treatment)

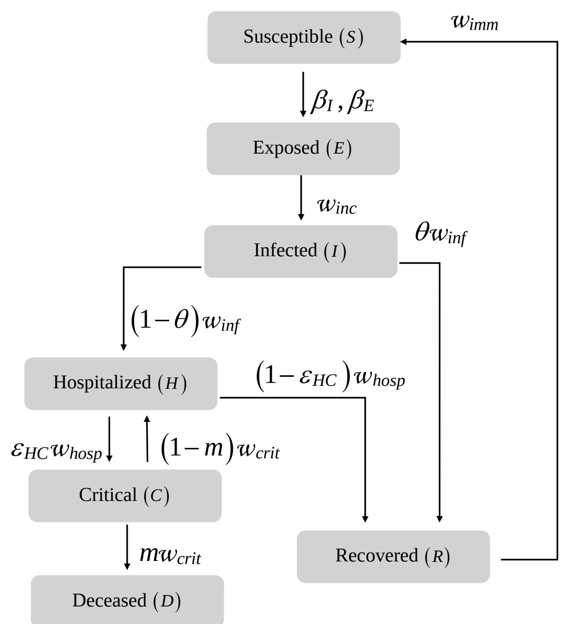

Figure 1: SEIR-HCD flow diagram

is

$$Y_t = (\mu_S(t) + \mu_E(t) + \mu_R(t)) \cdot r. \tag{3}$$

Table 1: Descriptions and approximate values of parameters in the SEIR-HCD model

Description, Symbol	**Potential Values**
Self-isolation index calculated by Yandex, a	$(0, 5)$
Probability to get infected when contacting with symptomatic individual, β_I	$(0, 1)$
Probability to get infected when contacting with asymptomatic individual, β_E	$(0, 1)$
Mean proportion of hospitalized cases requiring mechanical ventilation, ε_{HC}	$(0, 1)$
Mean proportion of infected individuals presenting no complications, θ	$(0, 1)$
COVID-19-related mortality rate, m	$(0, 0.5)$
Reverse to number of days between contact and becoming exposed, w_{inc}	$1/14 - 1/2; 1/\text{days}$
Reverse to duration in days for a severe case to escalate to a critical state, w_{hosp}	$1/5 - 1/4; 1/\text{days}$
Reverse to duration of the symptomatic period, w_{inf}	$1/14 - 2/5; 1/\text{days}$
Reverse to mean length of period with using of ventilation, w_{crit}	$1/20 - 1/10; 1/\text{days}$
Reverse to immunity period, w_{imm}	$1/150 - 1/60; 1/\text{days}$

Here r is positive constant determined the fraction of the average per capita income of the working population aimed at financing the prevention and treatment of the disease. Note that the coefficient r is determined by a combination of two factors: the fraction of the working-age population producing goods and services relative to the entire population, as well as the fraction of the tax determined by the social planner to react to the epidemic.

Expenditures on prevention and treatment reflect the social and economic costs of both direct costs associated with epidemic containment policies and indirect costs associated with a decrease in the labor force as morbidity increases. The costs of prevention and treatment will be determined using the following expression

$$C_t = (\mu_S(t) + \mu_E(t) + \mu_R(t)) \cdot c_p + \mu_I(t) \cdot c_i + \mu_H(t) \cdot c_h + \mu_C(t) \cdot c_c.$$

Here c_p is the cost paid by the uninfected (or considering themselves uninfected) part of the population due to compliance with restrictions and prevention; c_i – expenses for treatment of patients who do not require hospitalization; c_h – expenses for maintaining hospitalized patients; c_c – costs of maintaining critically ill patients on mechanical ventilation. Let $u(t) : [0, T] \to \mathrm{R}$ be a continuous function, meaning the cumulative loyalty of the population to isolation (compliance with restrictions). Put that $0 \le u(t) \le 1$ $\forall t$, where 0 means the complete loyalty of the population to antiviral measures, and 1 means the exact opposite case. Assume that $c_p = c_p(t)$ depends on the isolation strategy $u(t)$

$$c_p(t) = p_p(1 - u(t)),$$

where p_p is the price paid for prevention by an individual. The costs of treating patients who do not require hospitalization are determined by the price of treatment for an individual: $c_i = p_i$. The costs of maintaining hospitalized patients will be determined in the following form:

$$c_h(t) = \begin{cases} \dfrac{p_h}{H_{max} - \mu_H(t) + 1}, & \text{if } \mu_H(t) \le H_{max}, \\ (\mu_H(t) - H_{max}) p_h, & \text{otherwise}, \end{cases} \tag{4}$$

where p_h is the price of treatment for an individual who is hospitalized. The costs of maintaining critically ill patients on mechanical ventilation are determined in a similar way:

$$c_c(t) = \begin{cases} \dfrac{p_c}{C_{max} - \mu_C(t) + 1}, & \text{if } \mu_C(t) \le C_{max}, \\ (\mu_C(t) - C_{max}) p_c, & \text{otherwise}. \end{cases} \tag{5}$$

In (4),(5) constants H_{max}, C_{max} determine the maximum number of beds available in the region for the corresponding epidemiological groups. Thus, it is assumed that it is not profitable for the population to comply with restrictions due to additional economic costs, but the cost of each additional patient requiring hospitalization increases linearly.

2.3 Optimal control problem formulation

Assume that the rate of infection spread within the non-immune population depends on the isolation strategy chosen by the population and varies according to the rule:

$$\beta_I(t) = \beta_I^{min} + \left(\beta_I^{max} - \beta_I^{min}\right) u(t), \quad \beta_E(t) = \beta_E^{min} + \left(\beta_E^{max} - \beta_E^{min}\right) u(t). \tag{6}$$

Consider an optimal control problem with constraints. We will find the solution as a set of functions $\{\mu_{SEIRHCD}, u\}$ that provides a minimum of the functional

$$J = \int_0^T (C_t)^2 \mathrm{d}t, \tag{7}$$

where the notation $\mu_{SEIRHCD}(t)$ means the vector of state functions of the controlled system $\mu_{SEIRHCD}(t) := \{m_S(t), m_E(t), m_I(t), m_R(t), m_H(t), m_C(t), m_D(t)\}$. The optimal control problem is therefore formulated in the following manner: minimize the functional defined by (7) *subject to* the constraints imposed by the differential equations (1)–(2), while taking into consideration the expressions outlined in (3)-(6) and the following inequalities:

$$u(t) \leq 1, \quad \forall t \in [0, T], \tag{8}$$

$$u(t) \geq 0, \quad \forall t \in [0, T], \tag{9}$$

$$C_t(t) \leq Y(t), \quad \forall t \in [0, T]. \tag{10}$$

Here, inequalities (8), (9) allow one to remain within the chosen interval for measuring the rate of spread of the virus, and inequality (10) is a key restriction for the system under consideration, which determines that the level of costs associated with the fight against the virus does not exceed the resource which social planner can provide.

Note that the functions used are continuously differentiable with respect to u at least on open intervals where the corresponding function is defined. This guarantees the existence of a solution to the problem. Problem reformulation when describing other situations is possible, taking into account the fulfillment of the indicated condition.

3 Penalty functions algorithm for problem solving

Most constrained optimal control problems are too complex to obtain analytical solutions. Penalty methods (penalty function methods) are widely used for solving similar optimization problems (see works [19]–[21], and work by analyzing the convergence of the method [22]). The idea of the penalty method is to reformulate the entire functional taking into account restrictions in the form of a penalty and solve the unconstrained optimization problem. Note also that the penalty method is not the only one in its field. Also, to solve optimal control problems with constraints, the discretization method [23], the non-smooth Newton method [24, 25], and the control parameterization method [26] are used.

Thus, instead of the problem of minimizing the functional (7) the other optimal control problem is considered. The functional

$$J = \int_0^T \left((C_t)^2 + \frac{1}{\xi_1}(P_1)^2 + \frac{1}{\xi_2}(P_2)^2 + \frac{1}{\xi_3}(P_3)^2 \right) \mathrm{d}t \tag{11}$$

is minimized, where functions P_1, P_2, P_3 determine the penalties as

$$P_1(t) = \begin{cases} 0, \text{ if } u(t) \leq 1, \\ u(t) - 1, \text{ otherwize}, \end{cases} \quad P_2(t) = \begin{cases} 0, \text{ if } u(t) \geq 0, \\ -u(t), \text{ otherwize}, \end{cases} \quad P_3(t) = \begin{cases} 0, \text{ if } C_t(t) \leq Y_t(t), \\ C_t(t) - Y_t(t), \text{ otherwize}. \end{cases}$$

In (11) the parameters ξ_1, ξ_2, ξ_3 present the degree of influence of the corresponding restriction on the process under study. This substitution results in the formulation of an optimal control problem, which involves minimizing the functional (11) subject to the constraints represented by the system of differential equations (1) and (2). To solve the problem, let us obtain the conjugate system by the following way. Introduce smooth functions $\phi_i(t) : [0, T] \to \mathbb{R}$ and multiply $\phi_i(t)$ for all $i \in S, E, I, R, H, C, D$ with the corresponding equations in (1). Subsequently, vary the resultant Lagrangian with respect to the functions μ_i, where $i \in S, E, I, R, H, C, D$.

$$\begin{cases} \dfrac{d\phi_S}{dt} = (1 - a/5)(\phi_S - \phi_E)(\beta_I(t)\mu_I + \beta_E(t)\mu_E) - 2C_t\dfrac{\partial C_t}{\partial \mu_S} - 2\dfrac{P_3}{\xi_3}\dfrac{\partial P_3}{\partial \mu_S}, \\ \dfrac{d\phi_E}{dt} = (1 - a/5)\beta_E(t)\mu_S(\phi_S - \phi_E) + w_{inc}(\phi_E - \phi_I) - 2C_t\dfrac{\partial C_t}{\partial \mu_E} - 2\dfrac{P_3}{\xi_3}\dfrac{\partial P_3}{\partial \mu_E}, \\ \dfrac{d\phi_I}{dt} = (1 - a/5)\beta_I(t)\mu_S(\phi_S - \phi_E) + w_{inf}\,(\phi_I - \phi_H) + \\ \qquad\qquad + \theta w_{inf}\,(\phi_H - \phi_R) - 2C_t\dfrac{\partial C_t}{\partial \mu_I} - 2\dfrac{P_3}{\xi_3}\dfrac{\partial P_3}{\partial \mu_I}, \\ \dfrac{d\phi_R}{dt} = w_{imm}\,(\phi_R - \phi_S) - 2C_t\dfrac{\partial C_t}{\partial \mu_R} - 2\dfrac{P_3}{\xi_3}\dfrac{\partial P_3}{\partial \mu_R}, \\ \dfrac{d\phi_H}{dt} = w_{hosp}\,(\phi_H - \phi_R) + \varepsilon_{HC} w_{hosp}\,(\phi_R - \phi_C)\, 2C_t\dfrac{\partial C_t}{\partial \mu_H} - 2\dfrac{P_3}{\xi_3}\dfrac{\partial P_3}{\partial \mu_H}, \\ \dfrac{d\phi_C}{dt} = w_{crit}\,(\phi_C - \phi_H) + \mu w_{crit}\,(\phi_H - \phi_D) - 2C_t\dfrac{\partial C_t}{\partial \mu_C} - 2\dfrac{P_3}{\xi_3}\dfrac{\partial P_3}{\partial \mu_C}, \\ \dfrac{d\phi_D}{dt} = 0. \end{cases} \tag{12}$$

By varying the Lagrangian with respect to the control variable, we derive optimality conditions expressed in the form of

$$\begin{gathered} 2C_t\frac{\partial C_t}{\partial u} + 2\frac{P_1}{\xi_1}\frac{\partial P_1}{\partial u} + 2\frac{P_2}{\xi_2}\frac{\partial P_2}{\partial u} + 2\frac{P_3}{\xi_1}\frac{\partial P_3}{\partial u} + \\ + (1 - a/5)\mu_S(\phi_E - \phi_S)\big((\beta_I^{max} - \beta_I^{min})\mu_I + (\beta_E^{max} - \beta_E^{min})\mu_E\big) = 0. \end{gathered} \tag{13}$$

Finally, formulate an algorithm for solving the problem.

1. Put the parameters ξ_1, ξ_2, ξ_3 the large enough to neutralize the impact of penalty functions. For example, we can take $\xi_1 = \xi_2 = \xi_3 = 1000$ if the maximum of module of all penalty functions is less then tens.
2. Solve system (1),(2) with $u = 0$ to get μ_i functions corresponding to zero control.
3. Solve system (12) to get ϕ_i functions corresponding to obtained μ_i.
4. Solve (13) to get new control function $u(t)$.
5. Get new μ_i functions corresponding to obtained control.
6. Check the restrictions (8)-(10) perfoming. If $\exists t$ for which the $i-$th inequality (8)-(10) doesn't perform then $\xi_i := \xi_i/2$. Go to step 3 for new iteration.
7. When the inequalities (8)-(10) are performed, put current μ_i, u as the solution of the optimization problem.

Note that the presented algorithm is not exhaustive for all possible formulations and restrictions. Thus, if two or more chosen constraints are opposite, i.e. execution of one leads to automatic non-execution of the other, then the algorithm will obviously go in cycles. Countermeasures should be chosen based on the desired objectives. Perhaps the social planner will decide that one of the restrictions is insignificant or will relax it. Another solution is to consider a stopping criterion that analyzes the parameters ξ_i.

4 Numerical experiment

The performance of the algorithm will now be illustrated through the following example. This example utilizes actual data on the incidence of Covid-19 in the city of Novosibirsk over a 10-day period starting from July 12, 2020. Relevant statistics for the specified epidemiological groups can be accessed via an open resource [14]. The parameter values for model (1), as shown in Table 1, have been determined as averages over the designated time period, obtained through the solution of the inverse problem for model (1). The methodology for parameter estimation is detailed in [15]. The outcomes from solving the inverse problem can be found in a file available at the following link [16].

The size of the city's population, as well as the number of beds and ventilators available for the treatment are also known for the period under consideration: $N = 2780288$, $H_{max} = 800$, $C_{max} = 86$. Unfortunately, it isn't possible to compare the results obtained from modeling with actually collected statistics on morbidity for the same period, since the used measures and resources aimed at combating the epidemic are not known for sure. Thus, real data were used only to set the initial conditions and observe how the situation could develop under some scenarios. Assume that $r = 0.2$, $p_p = 0.1$, $p_i = 55$, $p_h = 110$, $p_c = 175$, $\beta_I^{max} = 4\beta_I$, $\beta_E^{max} = 4\beta_E$, $\beta_I^{min} = 0.25\beta_I$, $\beta_E^{min} = 0.25\beta_E$.

The parameters are chosen in such a way to describe a common situation. Prevention of disease costs orders of magnitude less than treatment. Moreover, when the disease progresses to a more severe form, the cost of treating such a patient increases several times. At the same time, the error in determining the rate of spread of the virus among different epidemiological groups can vary over a fairly wide range. Note that such an error in determining the contagiousness parameter is typical for the initial stages of a pandemic, when the incidence is just beginning to appear and it is not possible to make a qualitative assessment due to insufficient data. The rate r determines what proportion of the population's efforts is aimed at combating the epidemic.

Figs. 2-4 show the results of the computational experiment. Fig. 2 shows a comparison of the final control functions obtained in the absence of restrictions (8)-(10) and with them. In the absence of restrictions, the isolation strategy reflects the transition from compliance with anti-epidemiological measures to their absence, since the compliance leads to large expenses. At the same time, even with a very low cost of prevention (the population losses are lower), due to the large part of the population for which this prevention needs to be carried out, the costs of implementing anti-epidemic measures exceed the limits. Fulfillment of (8)-(9) leads to the shift of the paradigm of compliance with anti-viral restrictions, since even with the high cost of treatment, treating a small number of people becomes more profitable than spending resources on prevention for the majority of the population. This leads to a significant increase in morbidity (see Fig. 4).

Note that if we remove the limitation on expenses (10), but increase the cost of prevention (cost paid by the population for compliance with anti-viral measurements) as $p_p = 1$, then compliance with isolation and other antiviral measures becomes unprofitable even in the initial time period ($u(t) \equiv 1$ regardless of the presence or absence of restrictions in the form (8),(9)), which leads to the spread of the virus at maximum

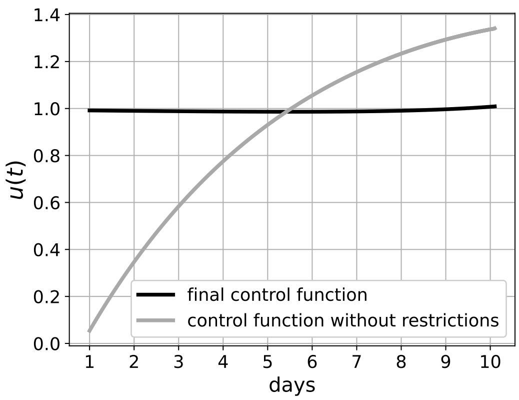

Figure 2: Isolation strategies obtained at the final iterations of the algorithm with and without restrictions

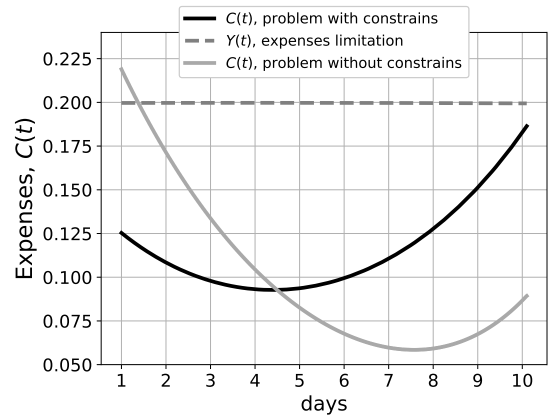

Figure 3: Expenses obtained at the final iterations of the algorithm with and without restrictions

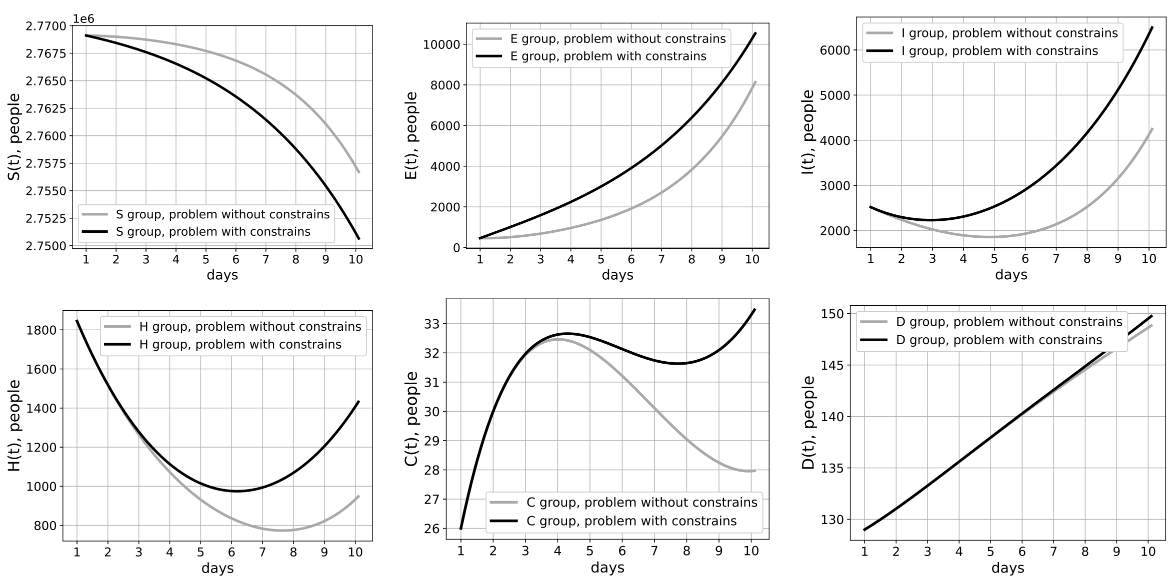

Figure 4: Number of people in each epidemiological group (S, E, I, H, C, D) for models with and without restrictions

speed.

Conclusion and discussion

The paper proposes a mathematical model for optimal control of the spread of the epidemic, taking into account restrictions on resources aimed at combating the virus. Note that the presented model reflects a general approach that can be used to solve similar problems with various constraints. The chosen SEIR-HCD epidemic propagation law can be changed to one that is suitable for describing the epidemiological situation (for example, with less detailed information on the distribution of the population among epidemiological groups, a simpler model can be used).

In the optimal control model, the behavior of the population is determined mainly by the choice of the cost functional and the restrictions imposed. Here we believe that the amount of funds allocated to combat the epidemic directly depends on decisions to comply with restrictive measures at previous points in time. There is also an assumption made here about the “anarchy” of the population - that is, the need to prevent the disease is caused not only by the need to reduce morbidity, but also by the reluctance of the population to comply with antiviral restrictions, since this increases their costs, both economic and social. In this case, model experiments have shown that investment in prevention of the population that has not yet been infected (or considers itself so) is a key factor in combating the spread of the disease. And the cost of prevention in fact determines the entire cost of fighting the epidemic. The results show that in the absence of strict restrictions (that is, when the population itself determines whether it should comply with antiviral restrictions), the optimal strategy is non-compliance with isolation. Note that the result obtained may turn out to be specific to the situation we have chosen, since the parameters chosen here corresponded to the beginning of the development of the epidemic, when only a small number of cases were recorded, and most of the population was not yet infected. Also, a change in paradigm leads to formulations with different functionals and restrictions, but the sequence of solving the problem remains the same.

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