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MEASUREMENT OF SUBMICRO-DEFORMATION OF OPTICAL ELEMENTS USING REFLECTIVE BACKGROUND ORIENTED SCHLIEREN TECHNIQUE

Mungalov A. S. [1], **Kochkin D. Y.**, **Pukhovoy M. V.**, **Zolotarev K. V.**, **Kabov O. A.**

Abstract This paper proposes to use a methodology for measuring deformations of solids that has important advantage over similar methods of this class, such as Phase measuring deflectometry and digital image correlation (DIC) based deflectometry which are known to require complex and labor-intensive calibration schemes, sometimes involving additional reference surfaces. We propose to use the reflective (moon-glade) background oriented schlieren (BOS) technique for measuring relative deformations of mirror surfaces. This method employs cross-correlation analysis of two reflected images of the background dot pattern from the mirror surface, similar to DIC-based deflectometry. The difference lies in the procedure of deformation field recovery, which is performed by solving the Poisson equation with the corresponding boundary conditions. A key advantage of the approach is the trivial calibration, requiring resolution in the background dot pattern plane and the mirror surface plane. Relative deformations of a steel specular substrate, induced by heating with a copper rod at its center, were measured. The results indicate that the resolution of this approach is approximately 50 nm. Measurement reliability was further validated by comparing the data with a commercial confocal optical sensor of the Micro-Epsilon, showing good agreement.

Key words: relative deformation, digital image correlation, Poisson equation, mirror surface, deflectometry.

AMS Mathematics Subject Classification: 78A55.

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1 Introduction

In various industries, including automotive parts manufacturing, display production, optical lens fabrication, telescope mirror and lens manufacturing and other product development three-dimensional (3D) measurement of freeform optical surfaces plays a critical role in quality control and functional evaluation. In synchrotron workstations, collimated synchrotron radiation is partially absorbed by the operational optical elements. The dissipated energy densities in these elements range from approximately 0.1 to 100 MW/m^2 [1-3]. To mitigate the thermal load on optical elements that directly form the image of the object under study in workstations, optical filters are employed to absorb less informative portions of the image spectrum. CVD diamond, beryllium, or glass carbon foils are commonly used in these elements [4-5] . These optical elements

[1] Corresponding Author.

experience the highest thermal load due to their absorption of a significant portion of the radiation in the working beam. Localized heating in the collimated beam can distort the flat section of the foil, significantly degrading the quality of the original beam. To minimize this phenomenon, deformations of flatness less than a few tenths of a micron must be achieved in a foil with a characteristic spatial size scale of 10 mm. Consequently, the flatness of the foil in an unloaded beam state should not exceed a couple of tens of nanometers. High reliability requirements are imposed on all optical elements on synchrotron workstations, with additional restrictions on the maximum allowable deformations [3, 5]. The requirements for permissible linear deformations caused by temperature gradients within the optical element can be as stringent as less than 0.1 μm over length scales of approximately 100 mm, as exemplified in the construction of the SKIF synchrotron facility near Novosibirsk. Even more stringent requirements for maintaining the necessary shape quality of optical elements are imposed on the mirrors of monochromators. Thus, in order to maintain the shape of synchrotron radiation beams, it is necessary to control the deformations of optical filters and mirrors. For this purpose, a spatial and easily implementable non-contact method for deformation measurements with very high resolution is required. Today, interferometry and deflectometry [6-8] is widely used to measure the surface of optical elements. Commercial interferometers are commonly employed in both laboratory and industrial settings. However, interferometers tend to be structurally complex and expensive. A relatively easy-to-implement approach is phase measuring deflectometry (PMD, type of deflectometry) [9]. This method involves reflecting a background pattern, typically consisting of sinusoidal fringes [10, 11] or a binary pattern [12], from the surface under study. A camera captures the reflection of the background pattern. Phase analysis of the captured images allows for the determination of x- and y-slope values. Ultimately, the surface is reconstructed by integrating the angles. Typically, PMD methods require the registration of a sinusoidal pattern in two perpendicular directions at least six times. Consequently, the process of deformation recovery is labor-intensive and susceptible to external disturbances. In addition, the calibration process is also non-trivial. Some limitations have been overcome in [13]. The authors proposed Real-Time PMD measurements and drawback of external disturbances was overcomed. In recent years, digital image correlation (DIC) has become increasingly popular for characterizing the deformations of solids under loads [14, 15]. Its key advantages include easy implementation, non-contact operation, adjustable resolution, and suitability for field measurements. To investigate the mechanical behavior of an object under realistic external loads, random dot pattern is applied to the object. Within the framework of this approach using cross-correlation of images of the object under load and without load it is possible to recover such parameters as shear modulus, shear strain distributions, magnitude of the maximum principle strain etc. [16-20]. Furthermore, in addition to two-dimensional cross-correlation approaches, there is a stereoscopic approach for measuring all the three displacement components of strains under loads [21]. Another approach that utilizes cross-correlation is the reconstruction of deformed surface topography. Methods based on specular and diffuse reflections followed by cross-correlation processing exist. Laser speckle based 3D profilometry [22, 23], which relies on diffuse reflection, is used for three-dimensional (3D) geometry reconstruction of various objects, finding applications in machine vision. The authors of [24] proposed a method of three-dimensional vision for measuring rapid full-field vibration patterns using a single high-speed camera based on a laser speckle structured light system. Specular reflection methods allow for the determination of the relative deformation of optical elements. In [25] the authors combined a deflectometry and DIC algorithm for dynamic deformation measurement of specular surfaces. They established a theoretical relationship between speckle deformation and displacement and proposed an experimental one-step calibration method. The calibration coefficient relating the local tilt angle to the displacement vector was determined using a precision rotation platform (BOCIC, MRS201) with an angular measurement resolution of 1" and repeatability of 2". More recently, the authors of [26] proposed single-shot dynamic speckle deflectometry (DSD), which locates the speckle shift pattern (SSP) by directly matching the captured single-shot speckle pattern with a computer-synthesized speckle pattern. While this method can measure the dynamic motion of a mirror surface, sophisticated calibration is still required to compare displacement and deformation. The authors of [27] proposed an approach in which the projection speckle pattern is reflected by the specular surface onto a screen, where the camera is oriented directly. This configuration addresses the issues of defocusing and external light effects. The beam-splitter and telecentric camera was added to the measurement scheme in [28]. The authors concluded that modified scheme allows the absolute surface shape and deformation at the same time to be measured, and has the advantages of simple measurement steps, fast response speed. We propose to use the reflective (moon-glade) background oriented schlieren (BOS) method which has been previously used to measure the free surface deformation of liquid layers [29-31]. In terms of the measurement process, this method is similar to deflectometry using the DIC algorithm, but the process of recovering the specular surface topography is different. Cross-correlation analysis is used to determine the displacement field. In turn, the relative deformations of the object are recovered by solving the Poisson equation with the corresponding boundary conditions. A key advantage of the approach over the reviewed works is the trivial callibration, requiring resolution in the background pattern plane and the surface of the object under study. In addition, an LED panel is used as the light source rather than a projector, making this measurement scheme more affordable. The reliability of measurements with the reflective (moon-glade) BOS is confirmed by comparison with the data obtained with a commercial confocal optical sensor Micro-Epsilon. This approach is a promising candidate for industrial-scale application for measuring relative deformations due to its simplicity in setup and image processing, trivial calibration, and very high resolution capability. This technique can also be extended to measure deformations relative to the horizontal plane by using backward DIC, as demonstrated in [26].

2 Experimental setup and methodology

2.1 Reflective (Moon-Glade) background oriented schlieren technique

The optical scheme of the reflective (Moon glade) BOS is presented in the Fig. 1. The method is based on analyzing of the background dot pattern reflected from the specular surface of a solid. Camera and dot pattern are placed above the investigated surface at an angle to the horizon so that the CCD camera captures the reflection of the background from the specular surface.

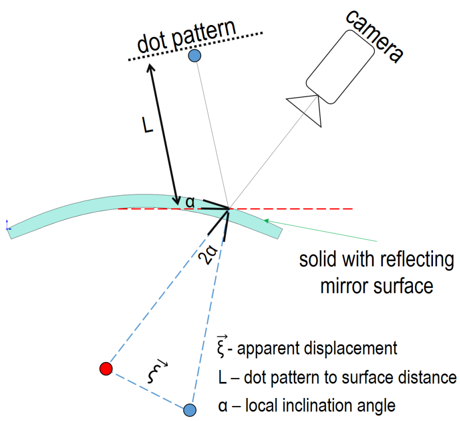

Figure 1: Optical scheme of reflective BOS.

Local curvature of the object leads to deviation of the initial ray path by an angle equal to twice the local angle of inclination of the object surface. Thus, the apparent displacement of the background dot can be represented as:

$$a_{back}\xi_{i,pix} = 2L\frac{\partial h}{\partial x_i}, \tag{1}$$

where x_i corresponding to the x and y coordinates along the reflecting surface, $\xi_{i.pix}$ - the dot displacement obtained by cross-correlation analysis in pix, a_{back} - image resolution (mm/pix) relative to the dot pattern plane, L - the distance from the dot pattern to the reflecting surface, h - local deformation. Since the displacement field ξ_i is obtained relative to the experimental data, the mixed derivatives of the deformation h do not turn to zero. Thus, it is necessary to minimize the functionality (2) which leads to the equation (3) and final Poisson equation on the deformation field (4):

$$\iint\Big[\Big(\frac{a_{back}}{2L}\xi_{x,pix} - \frac{\partial h}{\partial x}\Big)^2 + \Big(\frac{a_{back}}{2L}\xi_{y,pix} - \frac{\partial h}{\partial y}\Big)^2\Big]dxdy, \tag{2}$$

$$2\frac{\partial}{\partial x}\Big(\frac{a_{back}}{2L}\xi_{x,pix} - \frac{\partial h}{\partial x}\Big) + 2\frac{\partial}{\partial y}\Big(\frac{a_{back}}{2L}\xi_{y,pix} - \frac{\partial h}{\partial y}\Big) = 0, \tag{3}$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{a_{back}}{a_{surf}2L}\Big[\frac{\partial \xi_{x,pix}}{\partial x_{pix}} + \frac{\partial \xi_{y,pix}}{\partial y_{pix}}\Big], \tag{4}$$

a_{surf} - image resolution (mm/pix) relative to the reflecting surface plane. On the right-hand side of differential equation (4), the displacement ξ_i and the coordinates x_i are measured in pixels. The equation (4) is solved numerically by a finite-difference scheme of the second order of accuracy. The condition $h = 0$ is set on the boundaries of the investigated region. This condition is reasonable because the steel substrate used in the experiments is fixed around the periphery. The differential equation is also solvable if deformations are present on the boundary. In this case, the Neumann boundary conditions obtained from equation (1) is used:

$$\Big(\frac{\partial h}{\partial \vec{n}}\Big)_\Gamma = \frac{a_{back}\Big(\vec{\xi}_{n,pix}\Big)_\Gamma}{2L}, \tag{5}$$

where $\vec{n}$ - external normal to the boundary, Γ denotes the boundary. It is important to note that the next condition must be satisfied for the problem to be solvable:

$$\iint\limits_S \frac{a_{back}}{a_{surf}2L}\Big[\frac{\partial \xi_{x,pix}}{\partial x_{pix}} + \frac{\partial \xi_{y,pix}}{\partial y_{pix}}\Big]dxdy = \oint\limits_\Gamma \frac{a_{back}\Big(\xi_{n,pix}\Big)_\Gamma}{2L}dl, \tag{6}$$

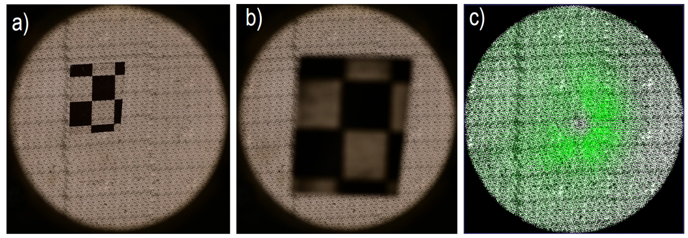

Figure 2: a) The image of reference target on the LED panel, b) on the steel specular substrate. c) the example of vector displacement field.

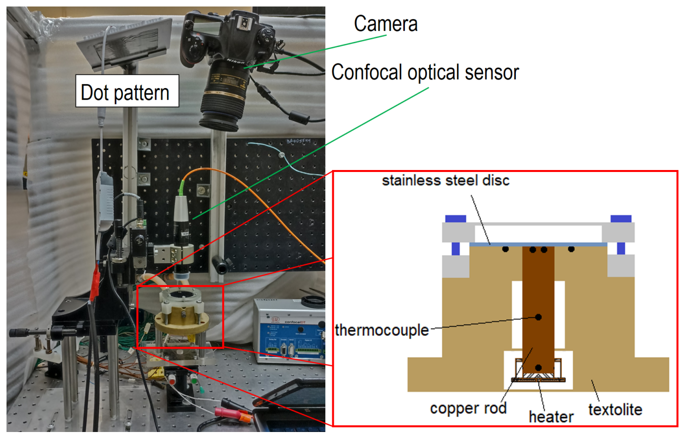

Figure 3: Experimental setup (left) and test section (right).

where l - length element. Thus, the field $h(x, y)$ is determined up to a constant factor in the general case.

The displacement field $\xi_{i,pix}$ is obtained using Digital-Image Correlation algorithm. The reference image, captured when the surface is flat (before heating the test plate), is consistently compared to images captured after the appearance of deformations. Each experimental image is divided into interrogation windows containing at least 10 background dots. Then the cross-correlation function is calculated, the maximum of which corresponding to the most probable displacement of the background dots within each interrogation window. Thus, each interrogation window contains one displacement vector. The size of the interrogation windows used in data analysis was 16x16 pixels, with a half-overlapping. For more detail about the cross-correlation method, refer to [32]. An example of a displacement vector field is shown in Fig. 2c.

As it was mentioned earlier, the advantage of this method is in the trivial calibration. A reference target is applied to the LED panel and specular substrate Figs. 2a,b with a known size of squares to determine the coefficient a_{back} and a_{surf} before the experiment. These two coefficients are sufficient to obtain h in real units.

2.2 Test section

Fig. 3 (left) shows an experimental setup for measuring thermal deformations caused by local heating. Experiments were carried out with a replaceable mirror substrate installed on the test section where it was heated in the center. To position the test section relative to the horizon, it was installed on the coordinate platform. Nikon D500 camera with an background oriented schlieren system was employed to visualize

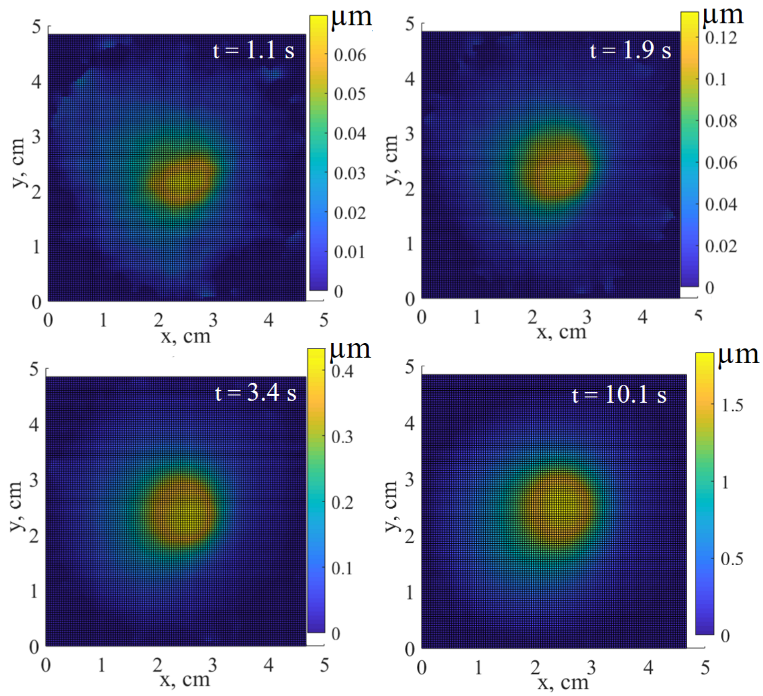

Figure 4: Relative deformation field of a specular substrate. The heating power is 46.3 W.

substrate deformations. The image resolution was 1920x1080. The frame rate was 60 frame/s. The inclination of the Led panel and the camera relative to the horizon was as 14° and 12° respectively. The distance from the background to the surface L was 43.2 cm. Micro-Epsilon IFC2461 controller with an IFS2405-1 confocal chromatic sensor was used to measure the instantaneous local deformation of the substrate. The confocal sensor was installed on the platform of the positioning system and oriented perpendicular to the surface of the substrate to measure the distance to it at a fixed point.

The test section base was a textolite plate with an embedded copper rod having a diameter of 12 mm (Fig. 3 right). A ceramic electrical heater was attached to the bottom edge of the copper rod. A substrate was installed on the base. The substrate was a stainless-steel disk with a diameter of 51 mm and a thickness of 1 mm. The substrate was treated by polishing the disk to a flat mirror surface. The method of treatment of the substrate provides the surface roughness (rms) of the order of 5 nm. The stainless-steel substrate is pressed to the base with a clamp. Thermal resistance between the substrate and the base was reduced by using thermal paste. The temperature of the surface of the copper rod, measured using the thermocouple (with an accuracy of about 0.1°C).

3 Results and discussion

The deformation profiles were measured for a thin stainless-steel disk, which was fixed on a heated base by pressing it at the edges with a clamp. Since the edges of the disk were firmly pressed to the substrate, local heating caused the specular substrate to bend due to linear thermal expansion. Fig. 4 shows three-dimensional relative deformation profiles of the disk caused by heating its center, the heating power is 46.3 W ($z=0$ is the initial state of the disk surface without deformation). The deformations are almost axisymmetric, and a slight asymmetry may be due to uneven pressing or thermal contact between the disk and the base.

Fig. 5 shows two-dimensional profiles of relative deformations of the disk along its diametrical line at heating powers and experimental times differing by almost an order of magnitude. In both cases, at the initial stage (Fig. 6, left), the deformation resembles a mound growing in the center, and at a higher heating power, the mound is more clearly visible. As heat spreads along the substrate due to thermal conductivity, the mound in the center becomes less distinct as the entire deformation grows (Fig. 5, right). At different heating powers, the temperature law of the heater differs (Fig. 6), which affects the behavior of thermal deformation of the substrate.

To validate the measurement results obtained using the reflective background oriented schlieren method, a comparison was made with data obtained using a confocal sensor measuring the relative change in the vertical position of the measured surface at a point in the center of the disk (Fig. 7). For more information about confocal optical sensor, refer to [33]. Measurements by two methods were carried out in different runs, but with the same experimental parameters. The data obtained by both methods at the central point of the steel mirror substrate due to its deformation caused by local heating are in good agreement.

4 Conclusion

The reflective (moon-glade) background oriented schlieren (BOS) technique is used for measuring steel mirror substrate deformations, induced by heating with a centrally positioned copper rod. This method relies on the analyzing images of a dot pattern reflected from the mirror surface under investigation. It involves a two-stage analysis process for deformation field reconstruction: initially, similar to digital image correlation (DIC) based deflectometry, a cross-correlation analysis of dot pattern images reflected from the original mirror surface and the deformed surface is performed. Subsequently, the Poisson's equation is numerically solved with appropriate boundary conditions. The main advantage of this method over existing approaches is trivial calibration, which consists in determining the resolution in the background pattern plane and the plane of the surface under study. The deformations of the steel mirror substrate were measured at different heating powers. The results indicate that the resolution of this approach is approximately 50 nm. Data obtained by the commercial confocal optical sensor of the Micro-Epsilon system were compared with reflective (moon-glade) BOS, showing good correspondence.

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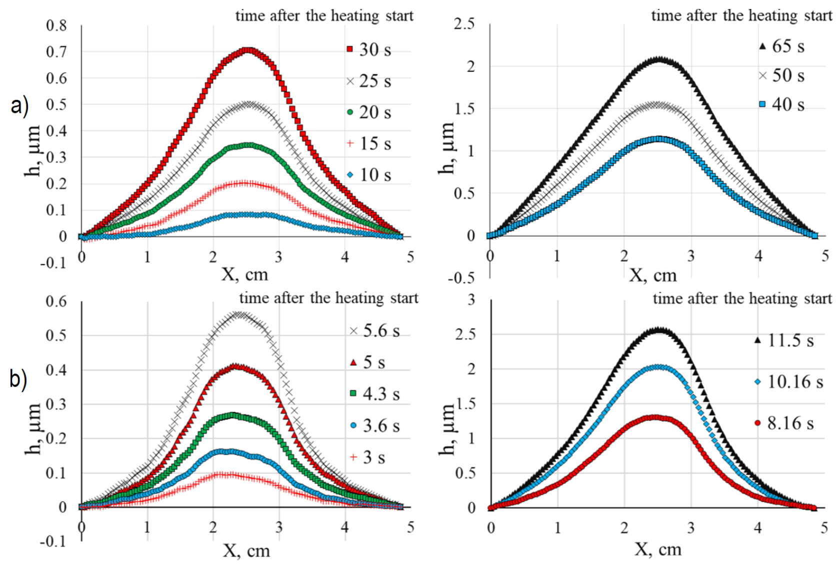

Figure 5: Deformation profiles of steel mirror substrate at different times after the heating start. a) The heating power is 4.2 W , b) the heating power is 46.3 W.

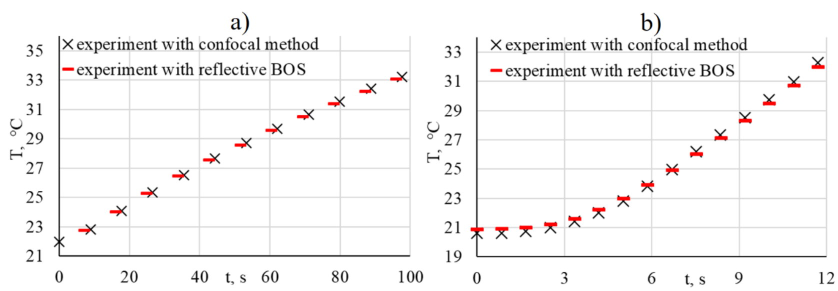

Figure 6: The heater temperature measurements under the center of the steel mirror substrate. a) The heating power is 4.2 W, b) the heating power is 46.3 W.

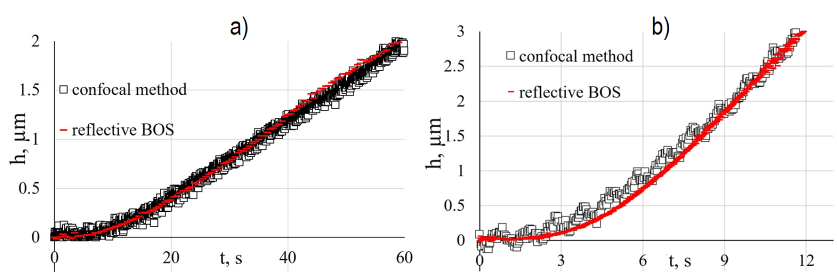

Figure 7: Comparison of the data obtained by reflective BOS and confocal optical method at the center of the steel mirror substrate. a) The heating power is 4.2 W, b) the heating power is 46.3 W.

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A.S. Mungalov,
Kutateladze Institute of Thermophysics SB RAS,
Lavrentiev av., 1, 630090 Novosibirsk, Russia,
Siberian Circular Photon Source "SKIF",
Boreskov Institute of Catalysis SB RAS,
Morskoy av., 2, 630090 Novosibirsk, Russia,
Email: a.mungalov@alumni.nsu.ru ,

D.Y. Kochkin,
Kutateladze Institute of Thermophysics SB RAS,
Lavrentiev av., 1, 630090 Novosibirsk, Russia,
Siberian Circular Photon Source "SKIF",
Boreskov Institute of Catalysis SB RAS,
Morskoy av., 2, 630090 Novosibirsk, Russia,

M.V. Pukhovoy,
Kutateladze Institute of Thermophysics SB RAS,
Lavrentiev av., 1, 630090 Novosibirsk, Russia,
Siberian Circular Photon Source "SKIF",
Boreskov Institute of Catalysis SB RAS,
Morskoy av., 2, 630090 Novosibirsk, Russia,

K.V. Zolotarev,
Siberian Circular Photon Source "SKIF",
Boreskov Institute of Catalysis SB RAS,
Morskoy av., 2, 630090 Novosibirsk, Russia,

O.A. Kabov,
Kutateladze Institute of Thermophysics SB RAS,
Lavrentiev av., 1, 630090 Novosibirsk, Russia.

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