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# RECURRENT NEURAL NETWORK LEARNING ALGORITHM-BASED EVENT-TRIGGERED OBSERVER OF THE PERMANENT MAGNET SYNCHRONOUS MOTOR

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**Abstract** In this paper, we propose a novel method to estimate the electrical angular velocity, the electrical angle, and the currents of the permanent magnet synchronous motor. A recurrent neural network learning algorithm is first developed to estimate the states of the permanent magnet synchronous motor. Then, an event-triggered state observer is designed for the recurrent neural network. This state observer robustly estimates state variables of the permanent magnet synchronous motor. A sufficient condition in terms of a convex optimization problem for the existence of the event-triggered state observer is established. In contrast with the abundance of state estimation methods based on time-triggered state observers where the measurements are always continuously available, the ones in this paper are updated when an event-triggered condition holds. Therefore, it lessens the stress on communication resources while can still maintain an estimation performance. Simulation results are provided to demonstrate the merit of the proposed method.

**Key words:** Permanent magnet synchronous motor (PMSM), event-triggered mehanism (ETM), event-triggered state obervers, linear matrix inequality (LMI).

**AMS Mathematics Subject Classification:** 34H05, 93B07, 93B51.

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## 1 Introduction

The information on state vectors of dynamical systems plays an important role in most engineering applications. It is often used to solve many practical problems such as state feedback control, system supervision, fault diagnosis of dynamic systems, and general diagnosis issues from available information [1], [11], [22], [23], [24], [36]. In particular, the authors of the work [1] used the information on state vectors of the nonlinear teleoperation system to solve the $H_\infty$ cost guaranteed integral sliding mode control for the synchronization problem, while the problem of designing a sliding mode observer for descriptor systems was considered in [11]. In [22], the authors solved the fault detection of time-delay systems by using functional observers, while state observers were designed in [23] for time-delay systems based on state transformations. The state estimation problem was investigated in [24] for fractional-order systems and in [36] for delayed stochastic neural networks. Nevertheless, due to technical or economic reasons, people usually use information state vector estimation instead of measuring the actual one. A typical example of this statement is the problem of estimating the speed and rotor position of permanent magnet synchronous motors (PMSMs) using a Kalman filtering technique [5], and using an improved square root unscented Kalman filter [43], which are brushless drives with all the properties required for servo applications [33]. In the PMSMs, the phase current must be a sinusoidal function of the rotor position. A high-resolution sensor is needed to obtain position information with appropriate resolution. Speed information may be derived from the position sensor or measured by a tachometer. These mechanical sensors increase the shaft inertia and dynamic friction, adding to the cost of the drive. They also need extra wiring beyond the cables required for supplying proper currents to the motor windings. These connections between the motor and the control system are often the source of an overall decrease in reliability. In order to reduce their cost and increase their reliability, PMSMs are not always equipped with mechanical sensors (rotor position and velocity). Instead, state observers are proposed to provide state variables of the PMSMs. This approach is very significant since electrical sensors tend to be cheaper and easier to maintain than mechanical ones.

Many methods are proposed in the literature to solve the problem of estimating the state vector of the PMSMs. For example, an identity state observer was proposed in [25], while a nonlinear speed observer is designed [32]. In [39], the authors proposed a mechanically sensorless full-state observer to solve the real-time observer-based control of a permanent magnet synchronous motor. Extended Kalman filters are implemented in [5] and [13] to estimate speed and rotor position. However, the above methods [5], [13], [25], [32], [39] did not consider the issue of the unknown load torque, which may lead to large estimation errors. To overcome this limitation, the authors of the work [42] proposed a nonlinear extended observer to estimate the state vector of a PMSM subject to an unknown load torque. Recently, there have been some interesting methods solving the state estimation problem of the PMSMs, for example, improved square root UKF [43], a nonlinear Luenberger approach for a non-observable system [35], sliding mode observer [26].

It is worth noting that all existing methods for estimating state vector of the PMSMs [5], [13], [25], [26], [32], [35], [39], [42], [43] were implemented based on time-triggered mechanisms, i.e., observer designs require system data for each sampling instant, which may lead to the wastage of communication resources in practical applications. So far, the methods in [5], [13], [25], [26], [32], [35], [39], [42], [43] have not been extended to event-triggered state estimation, which is useful in saving communication resource. Different from traditional state observers, event-triggered ones utilize only information from the output vector at triggering instants. Thus, they lessen the stress on communication resources while keeping the estimation performance. Many event-triggered state observers have been designed to solve the event-triggered state estimation. For example, an event-triggered extended state observer was designed in [18], while a dynamic event-triggered state observer was introduced in [20]. In [21], a discrete-time event-triggered state observer was designed to estimate the state of recurrent neural networks, while a state estimator that can successfully cope with event-based measurements was developed in [41]. In [19] and [38], the event-triggered-based stabilization problem was studied for onesided Lipschitz timedelay systems, and the neural network-based control system, respectively.

On the other hand, recurrent neural networks (RNNs) have attracted a lot of research attention in the literature [4], [15], [27], [44], [47]. In particular, the qualitative problem of dynamical neural networks was investigated in [4] and [15]. The high-order neural network structures were studied in [27], while the design of model predictive control systems by using an RNN was reported in [44]. In [47], the exponential stability problem was considered in for uncertain stochastic Hopfield neural networks. Since the structure of RNNs is more advantageous in designing event-triggered state observers than other nonlinear dynamical systems, in this paper, we aim to develop the RNN learning algorithm in [27], [44] to design event-triggered state observers for the PMSM. The main contributions of this paper are: (1) An RNN model is trained to estimate the states of the PMSM; (2) A new event-triggered state observer is designed to estimate the state vectors of the obtained RNN model; (3) An existence condition of such observer in terms of LMIs is established; and (4) Simulation results are obtained to demonstrate the applicability of the proposed method.

*Notation:* $A^T$ is the transpose of $A$. $||\cdot||$ denotes the Euclidean norm. $\mathbb{R}^n$ is the $n-$ dimensional linear vector space over $\mathbb{R}$. $P > 0$ means that $x^T Px > 0, \forall x \neq 0$. sym$\{A\}$ denotes $A + A^T$.

## 2 Preliminaries and problem statement

Consider the following permanent magnet synchronous motor [14]:

$$\begin{aligned} \dot{\omega}_r(t) &= \frac{3n_p^2}{2J}\Big(\psi_r + (L_d - L_q)i_d(t)\Big)i_q(t) - \frac{n_p}{J}s_L - \frac{1}{J}\mathcal{B}\omega_r(t), && (1)\\ \dot{\theta}_r(t) &= \omega_r(t), && (2)\\ \dot{i}_q(t) &= -\frac{R_s}{L_q}i_q(t) - \omega_r(t)\frac{L_d}{L_q}i_d(t) - \omega_r(t)\frac{\psi_r}{L_q} + \frac{1}{L_q}u_q(t), && (3)\\ \dot{i}_d(t) &= -\frac{R_s}{L_d}i_d(t) + \omega_r(t)\frac{L_q}{L_d}i_q(t) + \frac{1}{L_d}u_d(t), && (4) \end{aligned}$$

where $R_s$ is the stator resistance ($\Omega$), $u_d(t)$, $u_q(t)$, $i_d(t)$, $i_q(t)$, $L_d$ and $L_q$ are the $d-q$ axis stator voltages (V), currents (A) and inductances (Wb), respectively, $\psi_r$ is the amplitude of the permanent magnet flux linkage (Wb), $\omega_r(t)$ and $\theta_r(t)$ are the electrical angular velocity (rad/s) and the electrical angle (rad), $n_p$ is the number of pole pairs, $s_L$ is the load torque (N.m), $J$ and $B_b$ are the total moment of inertia (kg.$m^2$) and the viscous friction coefficient (Nm.s/rad).

Note that system (1)-(4) can be expressed into the following form

$$\dot{x}(t) = f(x(t), u(t), d(t)), \quad (5)$$

where $x(t) = \begin{bmatrix} \omega_r(t) \\ \theta_r(t) \\ i_q(t) \\ i_d(t) \end{bmatrix}$ is the state vector, $u(t) = \begin{bmatrix} u_q(t) \\ u_d(t) \end{bmatrix}$ is the measurable input vector, $d(t) = s_L$ is unknown input disturbances, $y(t) = \begin{bmatrix} \omega_r(t) \\ i_q(t) \\ i_d(t) \end{bmatrix}$ is the measurable output vector.

# 3 Main result

## 3.1 Approximating the permanent magnet synchronous motor by a recurrent neural networks

We first estimate the states of the PMSM (5) by the following RNN:

$$\begin{aligned} \dot{z}(t) &= f_{rnn}(z(t),u(t)) = Az(t) + \Theta_z\sigma(z(t)) + \Theta_u u(t),\ t \geq 0, &(6)\\ z(0) &= x_0, &(7)\\ \overline{y}(t) &= Cz(t), &(8)\end{aligned}$$

where $n = 4$, $m = 2$, $z(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $\overline{y}(t) = \begin{bmatrix} \overline{y}^1(t) & \overline{y}^2(t) \end{bmatrix}^T \in \mathbb{R}^{n+m}$, $\overline{y}^1(t) = \begin{bmatrix} \overline{y}_1 \\ \vdots \\ \overline{y}_n \end{bmatrix} = \begin{bmatrix} \sigma(z_1) \\ \vdots \\ \sigma(z_n) \end{bmatrix}$, $\overline{y}^2(t) = \begin{bmatrix} \overline{y}_{n+1} \\ \vdots \\ \overline{y}_{n+m} \end{bmatrix} = \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}$, where $\sigma(\cdot)$ is the activation function satisfying the following inequality

$$|\sigma_i(v_1) - \sigma_i(v_2)| \leq \bar{\sigma}_i|v_1 - v_2|,\ \forall v_1, v_2 \in \mathbb{R}, \tag{9}$$

where $\bar{\sigma}_i > 0$ for $i = 1, 2, \ldots, n$ are positive scalars. $A = \text{diag}\{-a_1, -a_2, \ldots, -a_n\}$, $\begin{bmatrix} \Theta_z \\ \Theta_u \end{bmatrix} = \Theta$, and $\Theta = \begin{bmatrix} \theta_1 & \ldots & \theta_n \end{bmatrix} \in \mathbb{R}^{(n+m)\times n}$ ($\theta_i = b_i \begin{bmatrix} w_{i1} & \ldots & w_{i(n+m)} \end{bmatrix}^T$, $a_i > 0$, $b_i$ are constants, for $i = 1, \ldots, n$, $j = 1, \ldots, n+m$, matrices $w_{ij}$ are the weight connecting from the $jth$ input to the $ith$ neuron, which will be optimized during training.

We can determine optimal weighs $a_i^*$ and $\theta_i^*$ for the RNN model (6) by solving the following ordinary least squares linear regression

$$(a_i^*, \theta_i^*) = \Lambda_i^* = \arg\min_{\Lambda_i} \frac{1}{2}\|S_i\Lambda_i - z_i\|^2, \tag{10}$$

where

$$S_i = \begin{bmatrix} [S_1]_i & \overline{y}_1^\top \\ [S_2]_i & \overline{y}_2^\top \\ \vdots & \\ [S_N]_i & \overline{y}_N^\top \end{bmatrix},\ z_i = \begin{bmatrix} f_i(s_1, u_1) \\ f_i(s_2, u_2) \\ \vdots \\ f_i(s_N, u_N) \end{bmatrix}, \Lambda_i = \begin{bmatrix} a_i \\ \theta_i \end{bmatrix}.$$

The following lemma provides an upper bound for the error between the state of the permanent magnet synchronous motor and the recurrent neural networks:

**Lemma 3.1.** *Assume that* $||d(t)|| \leq \bar{d}$, *where* $\bar{d}$ *is a positive number. If* $||f(z(t), u(t), 0) - f_{rnn}(z(t), u(t))|| \leq \bar{\omega}$, *then the following inequality holds:*

$$||e(t)|| \leq \frac{\frac{n_p}{J}\bar{d} + \bar{\omega}}{\gamma}(e^{\gamma t} - 1),\ t > 0, \tag{11}$$

*where* $e(t) = x(t) - z(t)$, $\bar{\omega}$ *is a positive number, and*

$$\gamma = \sqrt{\max\Big\{4M\frac{L_d^2}{L_q^2}, 2M\Big(\frac{9n_p^4(L_d-L_q)^2}{4J^2} + \frac{L_d^2}{L_q^2}\Big)\Big\}},$$

$M \in (0, \infty)$ *such that the states of the permanent magnet synchronous motor operate in a bounded region* $\mathcal{R} = \{z \in \mathbb{R}^4 | |z_i| \le M\}$.

***Proof:*** We have

$$\begin{aligned} ||\dot{e}(t)|| &= ||\dot{x}(t) - \dot{z}(t)|| = ||f(x(t), u(t), d(t)) - f_{rnn}(z(t), u(t))|| \\ &= ||f(x(t), u(t), d(t)) - f(z(t), u(t), 0) + f(z(t), u(t), 0) - f_{rnn}(z(t), u(t))||. \end{aligned} \tag{12}$$

On the other hand, for all $x, z \in \mathcal{R}$, the following inequality is satisfied:

$$\begin{aligned} &||f(x(t), u(t), d(t)) - f(z(t), u(t), 0)|| \\ &= \left\| \begin{bmatrix} \frac{3n_p^2(L_d-L_q)}{2J}x_4x_3 \\ 0 \\ -\frac{L_d}{L_q}x_1x_4 \\ \frac{L_q}{L_d}x_1x_3 \end{bmatrix} - \begin{bmatrix} \frac{3n_p^2(L_d-L_q)}{2J}z_4z_3 \\ 0 \\ -\frac{L_d}{L_q}z_1z_4 \\ \frac{L_q}{L_d}z_1z_3 \end{bmatrix} + \begin{bmatrix} -\frac{n_p}{J} \\ 0 \\ 0 \\ 0 \end{bmatrix} d(t) \right\|. \\ &\le \gamma||x(t) - z(t)|| + \frac{n_p}{J}\bar{d}. \end{aligned} \tag{13}$$

By using (12), (13) and inequality $||f(z(t), u(t), 0) - f_{rnn}(z(t), u(t))|| \le \bar{\omega}$, we obtain

$$||\dot{e}(t)|| \le \gamma||e(t)|| + \frac{n_p}{J}\bar{d} + \bar{\omega}. \tag{14}$$

Therefore, under the zero initial condition, the inequality (11) is obtained. The proof is completed.

## 3.2 Event-triggered state estimation problem for the recurrent neural networks

In the following, we consider that the measurement vector of (6) is not continuously implemented. Instead, it is only updated at triggering instants $\{s_k\}_{k\in\mathbb{N}}$ defined by the following dynamic event-triggered mechanism (DETM):

$$s_0 = 0,\ s_{k+1} = s_k + h\min\Big\{\xi \in \mathbb{N}^+ \mid \mathcal{H}(e_\rho(t), \rho(s_k)) > \gamma(s_k),\ h > 0\Big\}, \tag{15}$$

where $\mathcal{H}(e_\rho(t), \rho(s_k)) = \alpha[e_\rho(t)^T(t)\Xi e_\rho(t) - \mu\rho^T(s_k)\Gamma\rho(s_k)]$, $\rho(t) = z(t) - \hat{z}(t)$, $e_\rho(t) = \rho(s_k) - \rho(s_k + \xi h)$, $\xi \in \mathbb{N}$, $\alpha, \mu \in (0, \infty)$, $\Xi > 0$, function $\gamma(t)$ satisfies the following condition

$$\dot{\gamma}(t) = -\zeta\gamma(t) + \mu\rho^T(s_k)\Gamma\rho(s_k) - e_\rho^T(t)\Gamma e_\rho(t), \tag{16}$$

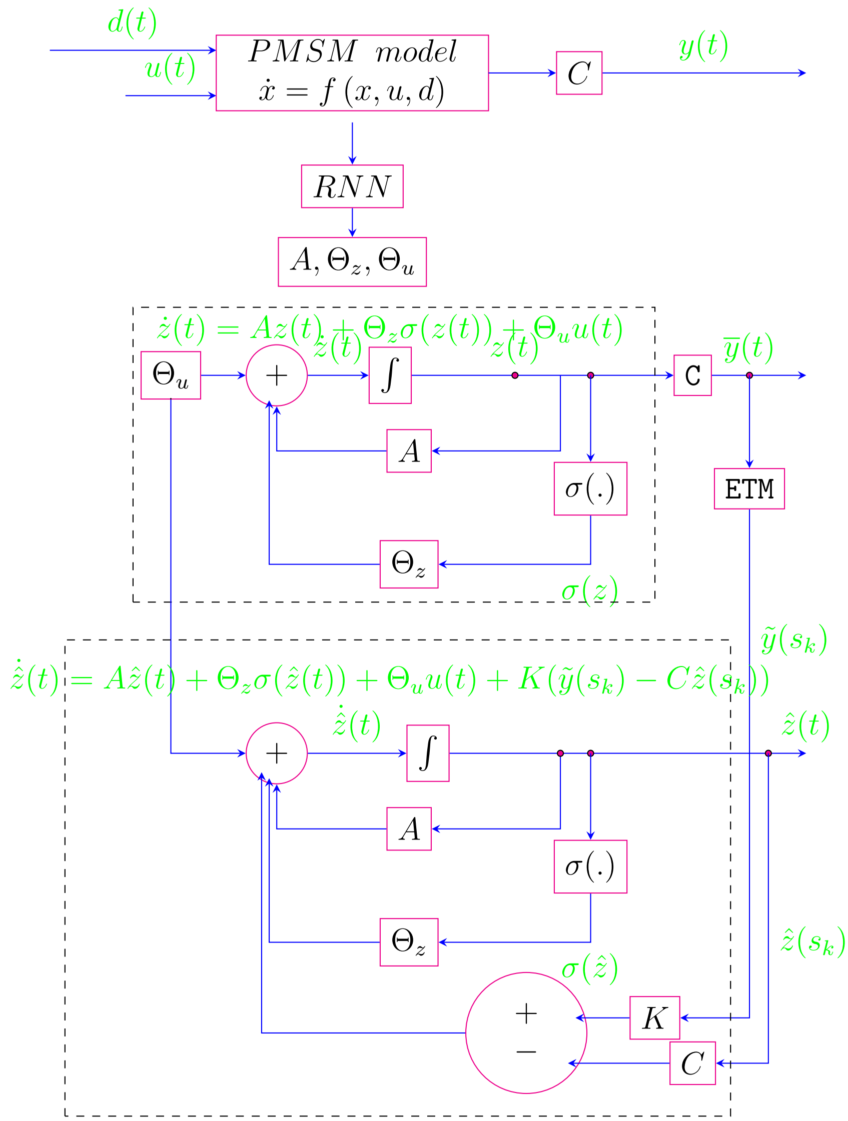


Figure 1: Schematic of an event-triggered state observer based on the DETM.

with $\zeta \in (0, \infty)$ and $\gamma(0) = 0$.

The event-triggered state observer based on the DETM (15) to estimate the state vector of the RNN model (6) is as below:

$$\dot{\hat{z}}(t) = A\hat{z}(t) + \Theta_z\sigma(\hat{z}(t)) + \Theta_u u(t) + K(\tilde{y}(s_k) - C\hat{z}(s_k)),\ t \in [s_k, s_{k+1}), \tag{17}$$

where $\hat{z}(t) \in \mathbb{R}^n$ is the estimate of $z(t)$, $K$ is the gain matrix to be designed. Denoting $\rho(t) = z(t) - \hat{z}(t)$, $\tau(t) = t - s_k - rh,\ t \in I_r$, one gets

$$\begin{aligned} \dot{\rho}(t) &= A\rho(t) + \Theta_z\sigma_{z\hat{z}}(t) - KC\rho(t - \tau(t)) - KCe_\rho(t), \\ & \quad t \in [s_k + \varepsilon_k, s_{k+1} + \varepsilon_{k+1}), \qquad (18) \\ \rho(s) &= \rho(0),\ s \in [-h, 0], \qquad (19) \end{aligned}$$

where $\sigma_{z\hat{z}}(t) = \sigma(z(t)) - \sigma(\hat{z}(t))$.

**Remark 1.** *The event-triggered observer (17) uses DETM (15), which depends on discrete supervision $\mathcal{H}(e_\rho(t), \rho(s_k)) > \gamma(s_k)$. It is clear from (15) that the minimum triggering interval $[s_k, s_{k+1})$ is one sampling period $h > 0$. Therefore, for DETM (15), the Zeno behaviour is excluded.*

We will determine the gain matrix $K$ such that system (18) is asymptotically stable.

**Theorem 3.1.** *Assumed that inequality $e^{\varepsilon\eta h} < \alpha\varepsilon + 1$ holds, where $\mu, \varepsilon, \alpha$ are positive scalars and $\eta$ is the smallest integer number satisfying $h \leq s_{k+1} - s_k \leq \eta h$. Given $\vartheta > 0$, (18) is asymptotically stable if there exist $\mathcal{P} > 0$, $\mathcal{Q} > 0$, $\mathcal{R} > 0$, $\Gamma > 0$, $Z$, $X$, non-singular $S$, positive scalars $\delta$, such that the following LMIs are feasible for $\theta \in \{0, 1\}$:*

$$\Delta^\star(\theta) = \begin{bmatrix} \Delta(\theta) & \nabla \\ * & -\delta I_n \end{bmatrix} < 0, \tag{20}$$

$$\Phi = \begin{bmatrix} diag(\mathcal{R}, \mathcal{R}) & Z \\ * & diag(\mathcal{R}, \mathcal{R}) \end{bmatrix} > 0, \tag{21}$$

*where*

$$\begin{aligned}
\Delta(\theta) &= \Delta_1(\theta) + \Delta_2(\theta) + \delta\bar{\sigma}_{\max}^2 \epsilon_1^T \epsilon_1, \bar{\sigma}_{\max} = \max\{\bar{\sigma}_1, \ldots, \bar{\sigma}_n\}, \\
\Delta_1(\theta) &= sym\{\Psi_\theta^T P \Gamma\} + \epsilon_1^T \mathcal{Q} \epsilon_1 - \epsilon_3^T \mathcal{Q} \epsilon_3 + h^2 \epsilon_4^T R_2 \epsilon_4 - \Lambda^T \Phi \Lambda \\
&\quad + \mu \epsilon_2^T \Gamma \epsilon_2 + \mu \epsilon_7^T \Gamma \epsilon_7 - sym\Big\{ \begin{bmatrix} \epsilon_1^T & \epsilon_4^T \end{bmatrix} \begin{bmatrix} \vartheta S \\ S \end{bmatrix} \epsilon_4 \Big\}, \\
\Delta_2(\theta) &= sym\Big\{ \begin{bmatrix} \epsilon_1^T & \epsilon_4^T \end{bmatrix} \begin{bmatrix} \vartheta S \\ S \end{bmatrix} A \epsilon_1 - \begin{bmatrix} \epsilon_1^T & \epsilon_4^T \end{bmatrix} \begin{bmatrix} \vartheta X \\ X \end{bmatrix} C \epsilon_2 - \\
&\quad - \begin{bmatrix} \epsilon_1^T & \epsilon_4^T \end{bmatrix} \begin{bmatrix} \vartheta X \\ X \end{bmatrix} C \epsilon_7, \\
\Psi_\theta &= \begin{bmatrix} \epsilon_1^T & \theta h \epsilon_5^T + (1-\theta) h \epsilon_6^T \end{bmatrix}^T, \Gamma = \begin{bmatrix} \epsilon_4^T & (\epsilon_1^T - \epsilon_3^T) \end{bmatrix}^T, \\
\Lambda &= \begin{bmatrix} \Lambda_1 & \Lambda_2 & \Lambda_3 & \Lambda_4 \end{bmatrix}^T, \Lambda_1 = \begin{bmatrix} (\epsilon_1 - \epsilon_2)^T \end{bmatrix}, \\
&\quad \Lambda_2 = \begin{bmatrix} \sqrt{3}(\epsilon_1 + \epsilon_2 - 2\epsilon_5)^T \end{bmatrix}, \\
&\quad \Lambda_3 = \begin{bmatrix} (\epsilon_2 - \epsilon_3)^T \end{bmatrix}, \Lambda_4 = \begin{bmatrix} \sqrt{3}(\epsilon_2 + \epsilon_3 - 2\epsilon_6)^T \end{bmatrix}, \\
\nabla &= \begin{bmatrix} \epsilon_1^T & \epsilon_4^T \end{bmatrix} \begin{bmatrix} \vartheta S \\ S \end{bmatrix} \times \Theta_z \begin{bmatrix} I_n & 0_{n\times n} \end{bmatrix}, \\
&\quad \epsilon_i = \begin{bmatrix} \epsilon_i^1 & \epsilon_i^2 \end{bmatrix} \in \mathbb{R}^{n\times(7n)}, i = 1, \ldots, 7, \\
&\quad \epsilon_i^1 = \begin{bmatrix} 0_{n\times(i-1)n} & I_n \end{bmatrix}, \epsilon_i^2 = \begin{bmatrix} 0_{n\times(7-i)n} \end{bmatrix}.
\end{aligned}$$

*The observer gain matrix $K$ is obtained as*

$$K = S^{-1} X. \tag{22}$$

***Proof:***

We denote $\tilde{e}(t) = \begin{bmatrix} \rho^T(t) & s_{t-h}^t \rho^T(s) ds \end{bmatrix}^T$ and consider the following Lyapunov function:

$$V(t) = \gamma(t) + \tilde{e}^T(t) \mathcal{P} \tilde{e}(t) + s_{t-h}^t \rho^T(s) \mathcal{Q} \rho(s) ds + h s_{-h}^0 s_{t+\eta}^t \dot{\rho}^T(s) \mathcal{R} \dot{\rho}(s) ds. \tag{23}$$

In light of the proof of Lemma 4 in [21], we can prove that $\gamma(t) \geq 0,\ \forall t > 0$. Thus, $V(t) \geq 0,\ \forall t > 0$. Taking derivative of $V(t)$ in $t$, we obtain

$$\begin{aligned}\dot{V}(t) \;=\; & -\lambda\gamma(t) + \mu(\rho(t-\tau(t)) + \epsilon_\rho(t))^T\Gamma(\rho(t-\tau(t)) + \epsilon_\rho(t)) + 2\zeta^T(t)\Psi_\theta^T P\Gamma\zeta(t) + \\ & +\zeta^T(t)[\epsilon_1^T\mathcal{Q}\epsilon_1 - \epsilon_3^T\mathcal{Q}\epsilon_3]\zeta(t) + h^2\zeta^T(t)(\epsilon_4^T\mathcal{R}\epsilon_4)\zeta(t) - \\ & -hs_{t-\tau(t)}^{t}\dot{\rho}^T(s)\mathcal{R}\dot{\rho}(s)ds - hs_{t-h}^{t-\tau(t)}\dot{\rho}^T(s)\mathcal{R}\dot{\rho}(s)ds, \end{aligned} \tag{24}$$

where

$$\begin{aligned} & \zeta(t) = \begin{bmatrix} \zeta_1(t) & \zeta_2(t) & \zeta_3(t) & \zeta_4(t) \end{bmatrix}^T,\ \zeta_1(t) = \begin{bmatrix} \rho^T(t) & \rho^T(t-\tau(t)) \end{bmatrix} \\ & \zeta_2(t) = \begin{bmatrix} \rho^T(t-h) & \dot{\rho}^T(t) \end{bmatrix},\ \zeta_3(t) = \begin{bmatrix} \frac{1}{\tau(t)}s_{t-\tau(t)}^{t}\rho^T(s)ds \end{bmatrix}, \\ & \zeta_4(t) = \begin{bmatrix} \frac{1}{h-\tau(t)}s_{t-h}^{t-\tau(t)}\rho^T(s)ds & e_\rho^T(t) \end{bmatrix}. \end{aligned} \tag{25}$$

Now, by employing the Wirtinger-based integral inequality [40], the reciprocally convex combination inequality [34], the Cauchy matrix inequality and the Schur Complement Lemma [9], one gets

$$\dot{V}(t) \leq \zeta^T(t)\Delta^\star(\theta)\zeta(t), \tag{26}$$

where $\theta \in (0,1)$.

Since $\Delta^\star(\theta)$ is convex with respective to $\theta$, $\Delta^\star(\theta) < 0\ \forall\theta \in \{0,1\}$ implies $\Delta^\star(\theta) < 0\ \forall\theta \in (0,1)$. Thus, (18) is asymptotically stable. The proof is completed.

## 4 Simulation results

The trajectory of the PMSM ($x_1(t) = \omega_r(t)$, $x_2(t) = \theta_r(t)$, $x_3(t) = i_q(t)$, $x_4(t) = i_d(t)$) and the RNN ($z_1(t)$, $z_2(t)$, $z_3(t)$, $z_4(t)$) are shown in Figure 2, Figure 4, Figure 6, Figure 8, while the errors between $x_i(t)$ and $z_i(t)$ ($i = 1,2,3,4$) are shown in Figure 3, Figure 5, Figure 7, Figure 9. Figure 10 depicts intervals of ETM (12). Figure 11, Figure 13, Figure 15, Figure 17 plot the trajectory of the RNN and its estimation, while the errors between $z_i(t)$ ($i = 1,2,3,4$) and their estimations are shown in Figure 12, Figure 14, Figure 16, Figure 18. From the above figures, we see that the proposed method can estimate the trajectory of the PMSM.

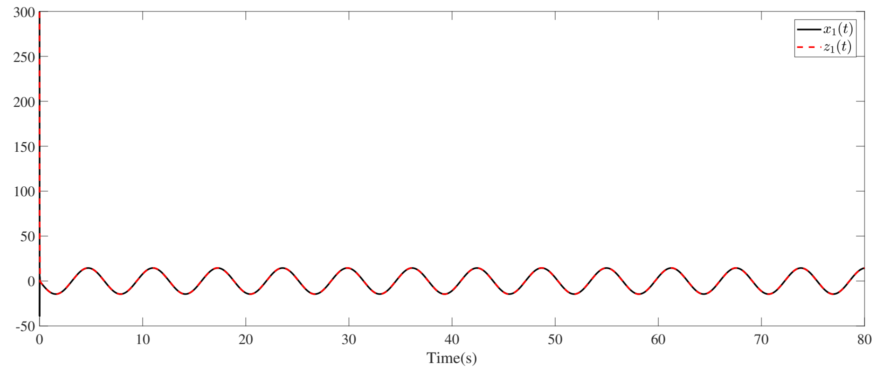


Figure 2: $x_1(t)$ and $z_1(t)$

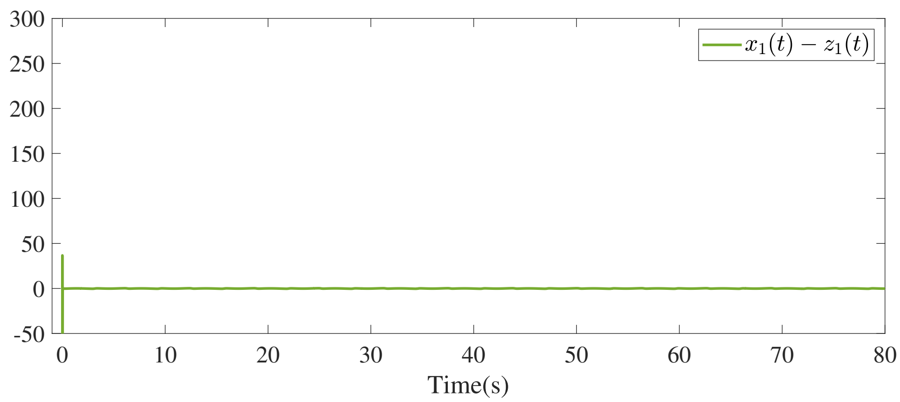


Figure 3: $x_1(t) - z_1(t)$

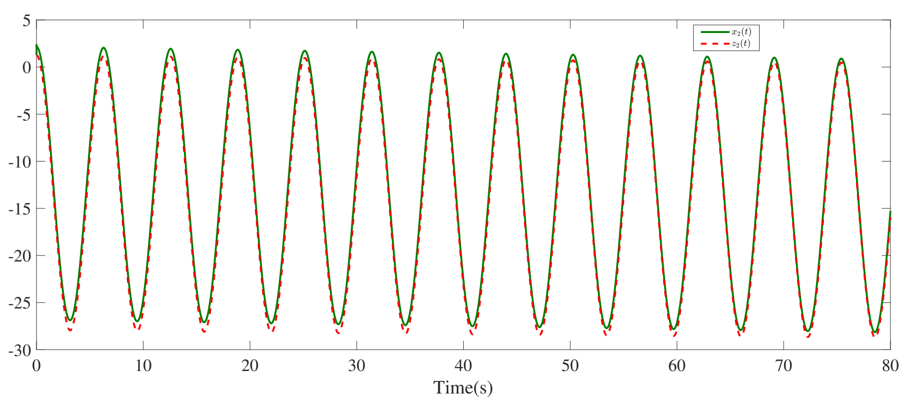


Figure 4: $x_2(t)$ and $z_2(t)$

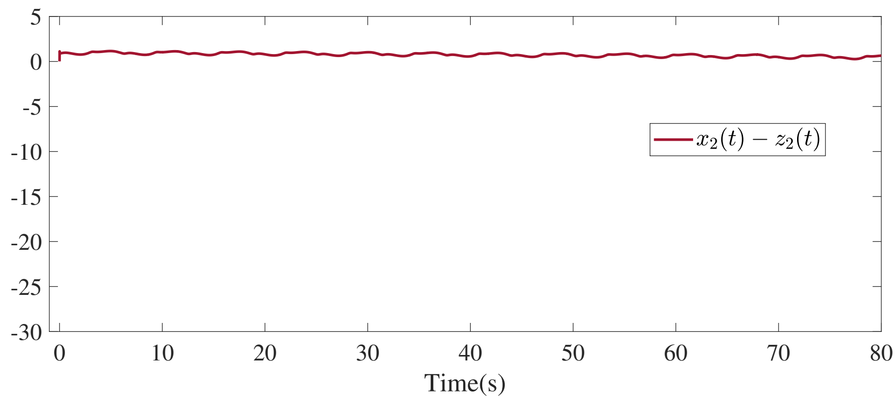


Figure 5: $x_2(t) - z_2(t)$

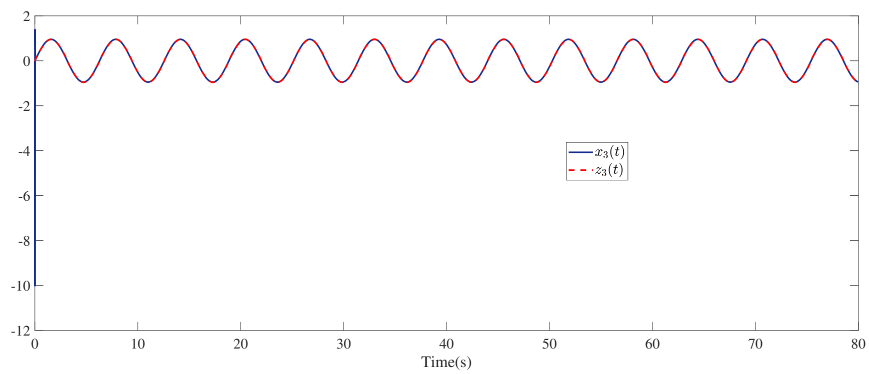


Figure 6: $x_3(t)$ and $z_3(t)$

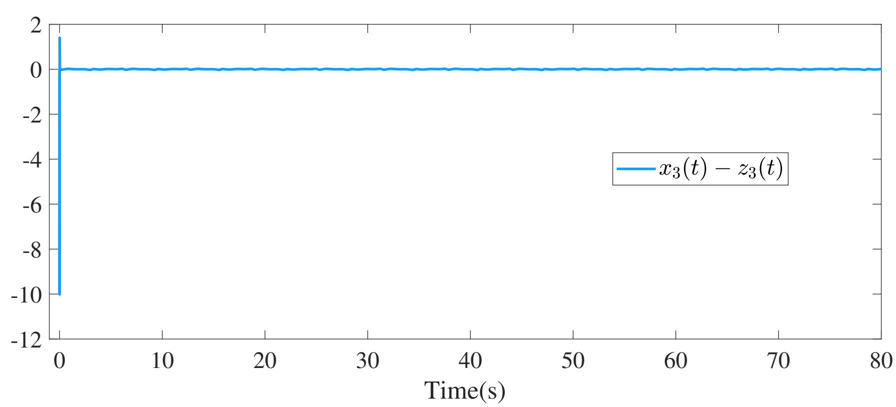


Figure 7: $x_3(t) - z_3(t)$

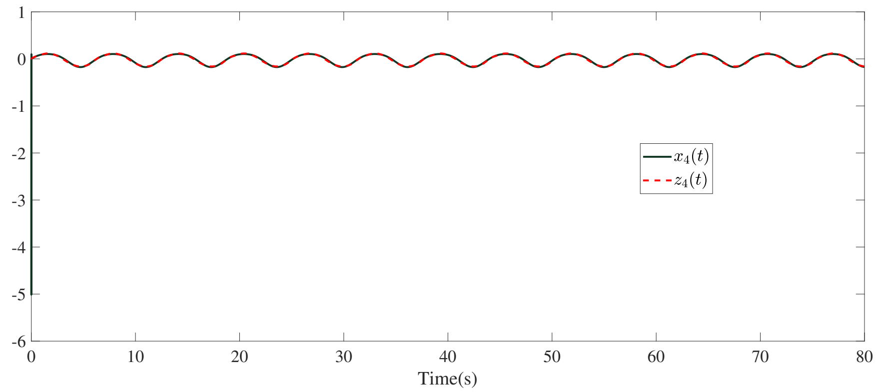


Figure 8: $x_4(t)$ and $z_4(t)$

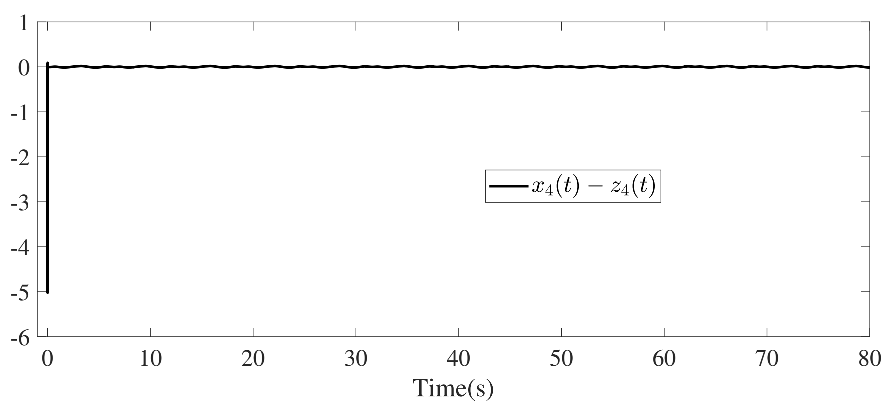


Figure 9: $x_4(t) - z_4(t)$

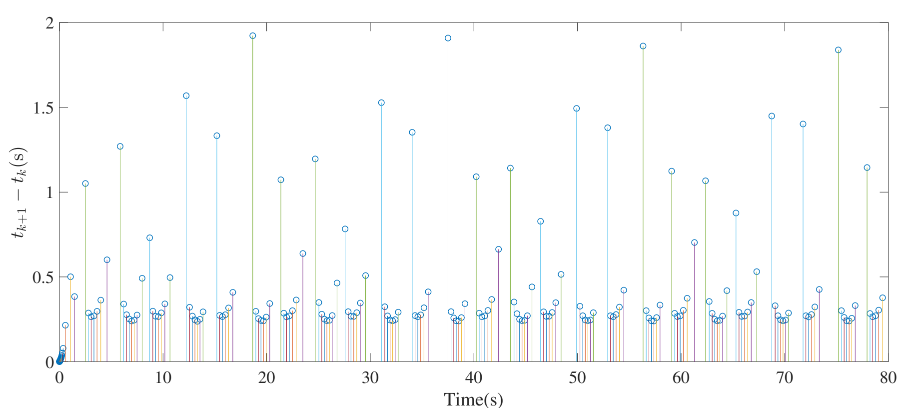


Figure 10: Intervals of ETM (12)

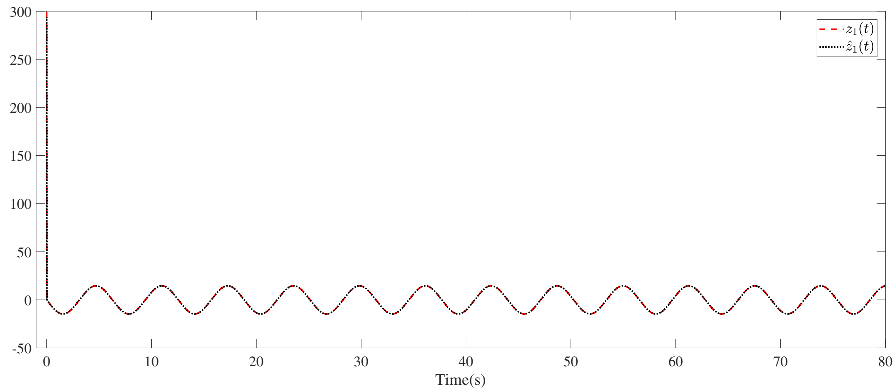


Figure 11: $z_1(t)$ and its estimation

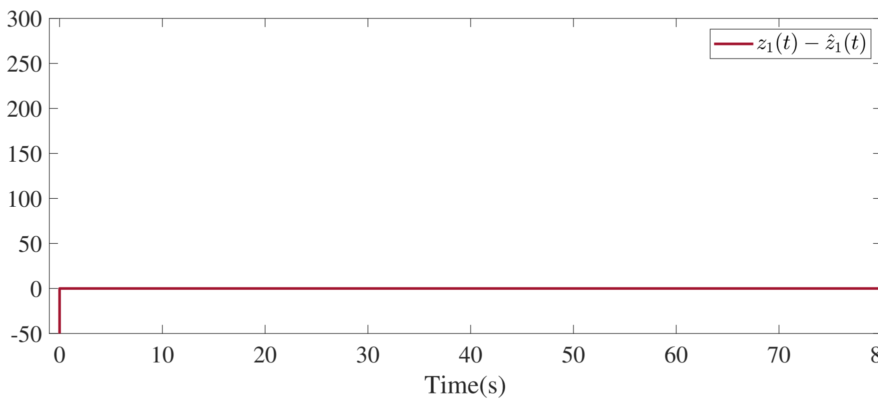


Figure 12: $z_1(t) - \hat{z}_1(t)$

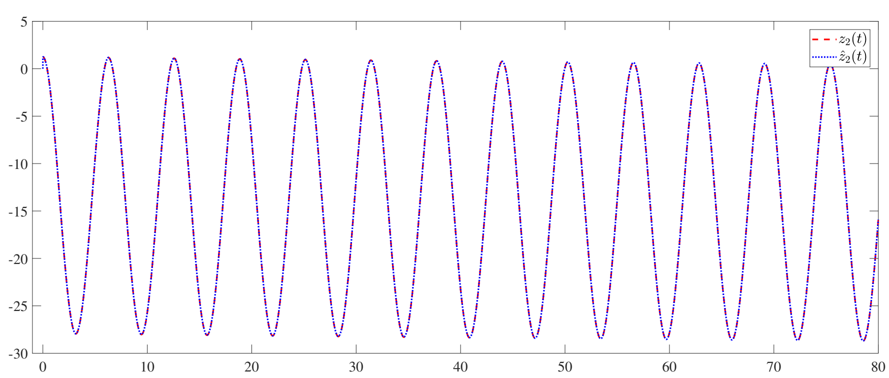


Figure 13: $z_2(t)$ and its estimation

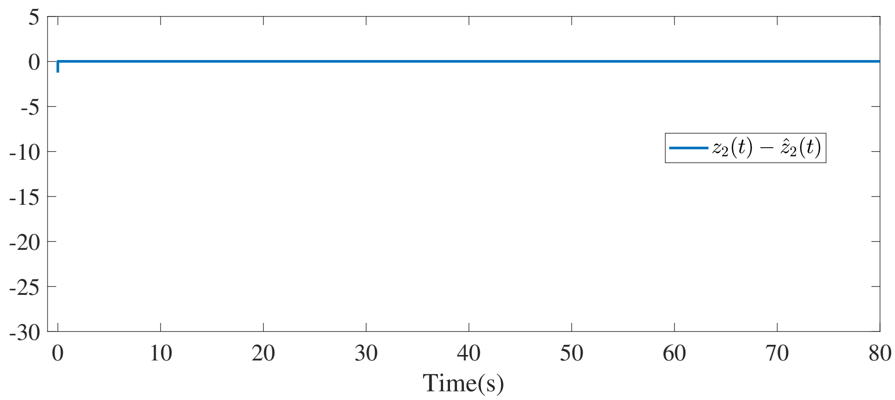


Figure 14: $z_2(t) - \hat{z}_2(t)$

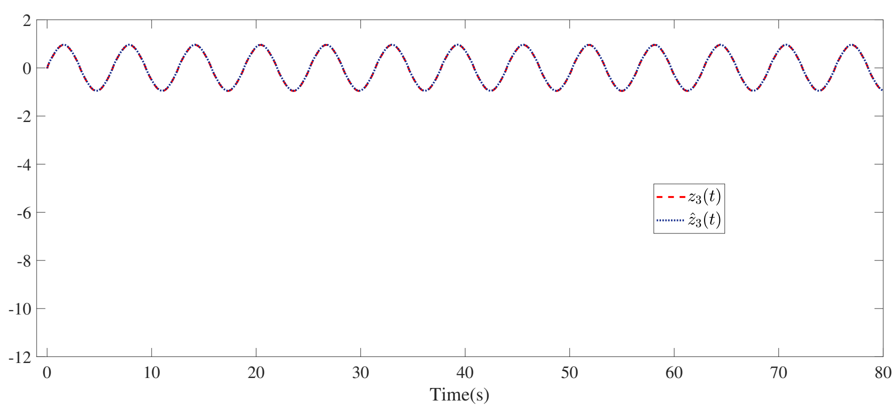


Figure 15: $z_3(t)$ and its estimation

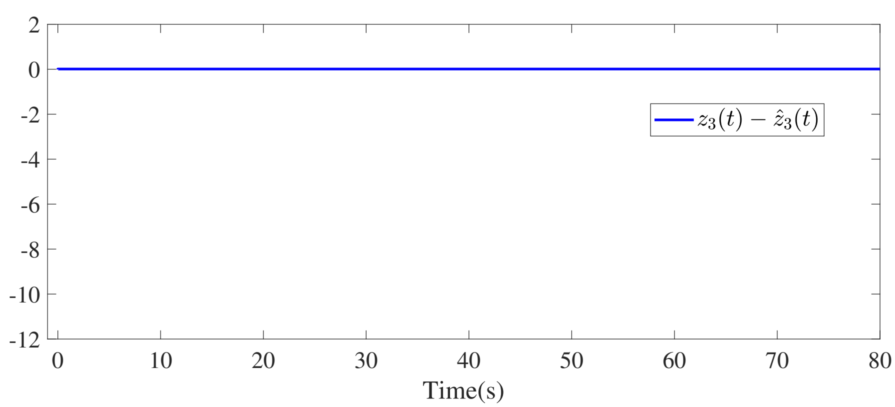


Figure 16: $z_3(t) - \hat{z}_3(t)$

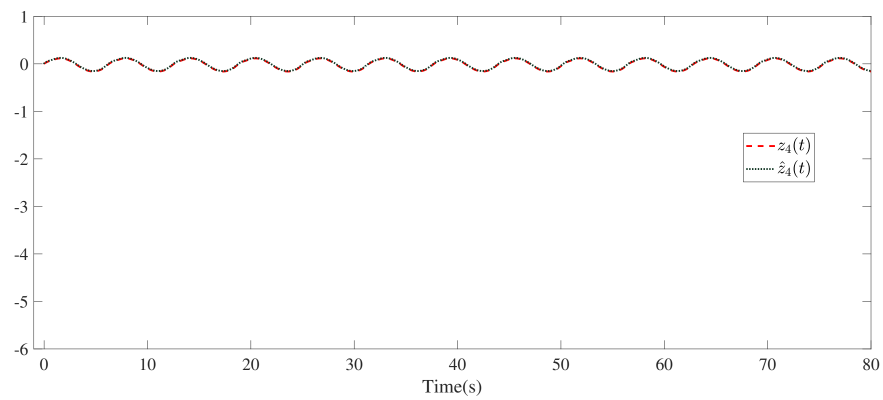


Figure 17: $z_4(t)$ and its estimation

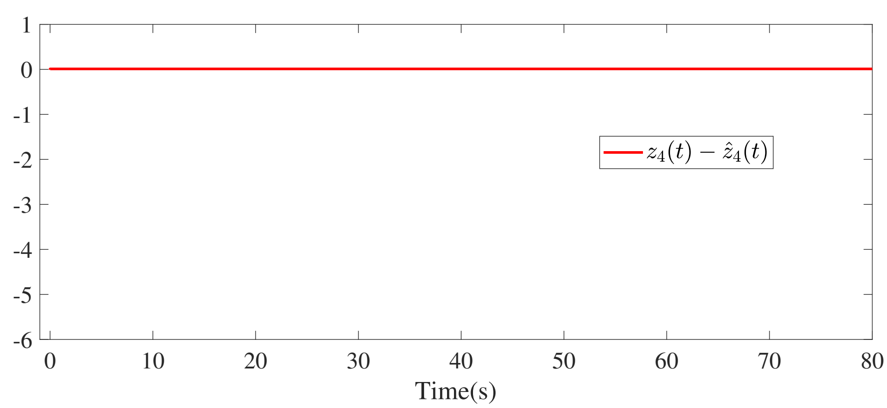


Figure 18: $z_4(t) - \hat{z}_4(t)$

## 5 Conclusion

We have solved the problem of estimating the electrical angular velocity, the electrical angle, and the currents of the permanent magnet synchronous motor. A RNN model which predicts the permanent magnet synchronous motor and a dynamic event-triggered state observer for this model have been derived. Simulation results have been provided to demonstrate the merit of the proposed method.

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